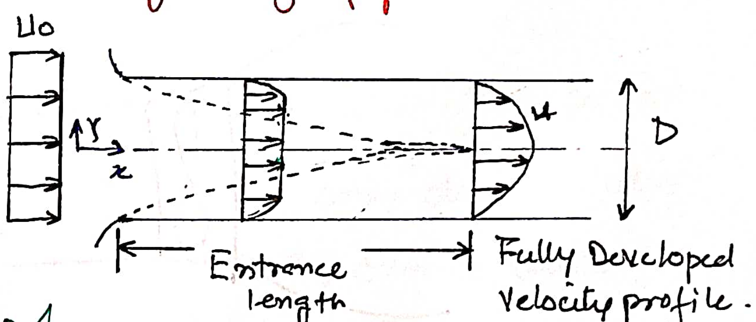
Pipe flow.

Laminar & Turbulent flows: A Laminar flow is one in which the fluid flows in layers. There is no macroscopic mixing of adjacent fluid layers. Consider a flow through a pipe. At Low Re, flow is Laminar but as Re increases, after a certain value of Re, the flow becomes turbulent. This turbulent flow is caused by small, high frequency velocity fluctuations superimposed on the mean velocity. For pipe flows, under normal conditions, transition to turbulence occurs at $Re \cong 2300$.

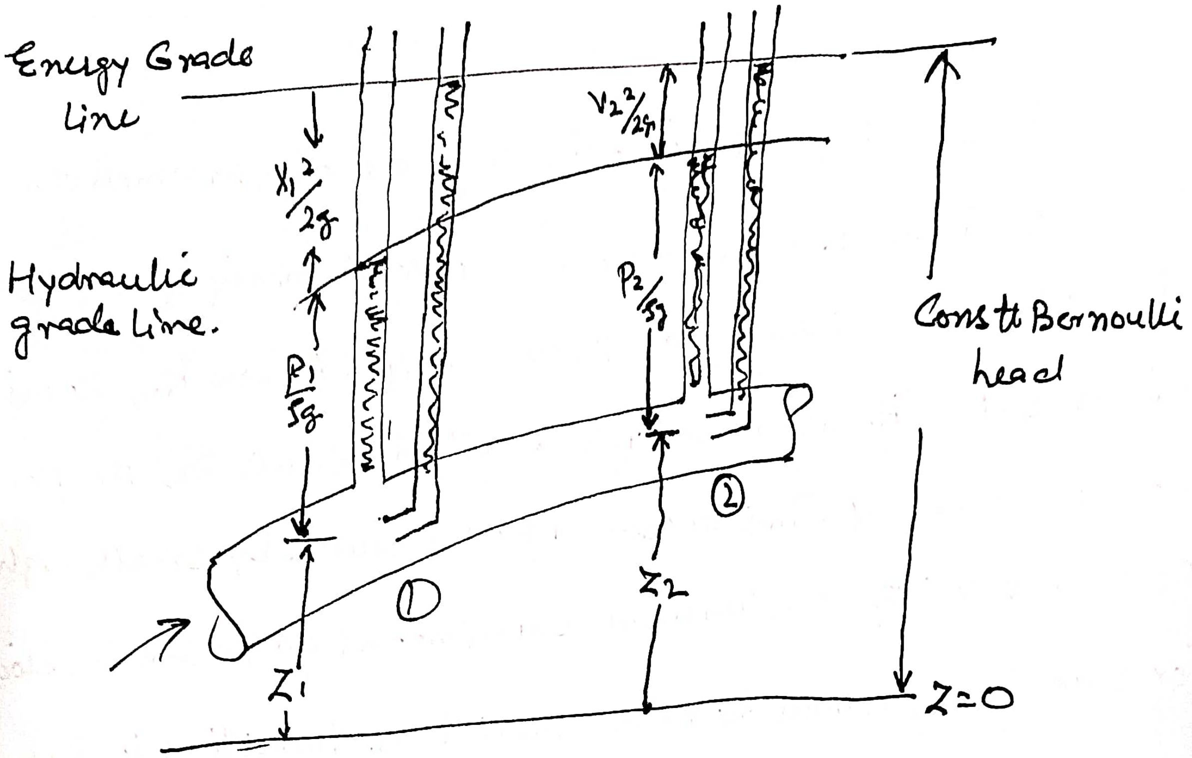
Flow in the entrance region of a pipe

Consider a Laminar flow with a uniform velocity U_o at entrance. Due to no-slip B.C. velo. at wall must be zero, A B.L. develops and grows until the B.L from the top & bottom wall meet.

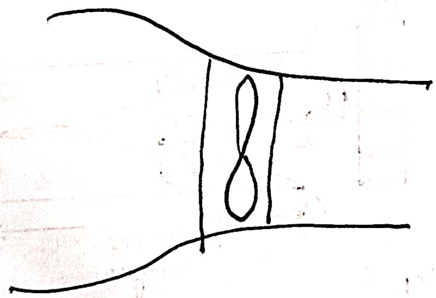


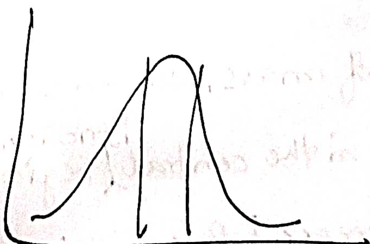
Due to conservation of mass, we can say that as the speed close to the wall is reduced, the speed in the central [frictionless] region must increase and the pressure must decrease [Bernoulli eq.]. The avg. velo. magnitude at any crossection $\bar{V} = \frac{1}{A}\int_{Area} u\,dA$ must be equal to U_o. When the velo. profile shape no longer changes with increasing distance x, the flow is called fully developed. For Laminar flow, entrance length L, is given by $\frac{L}{D} \simeq 0.06\,Re$.

Fully Developed Laminar Fl



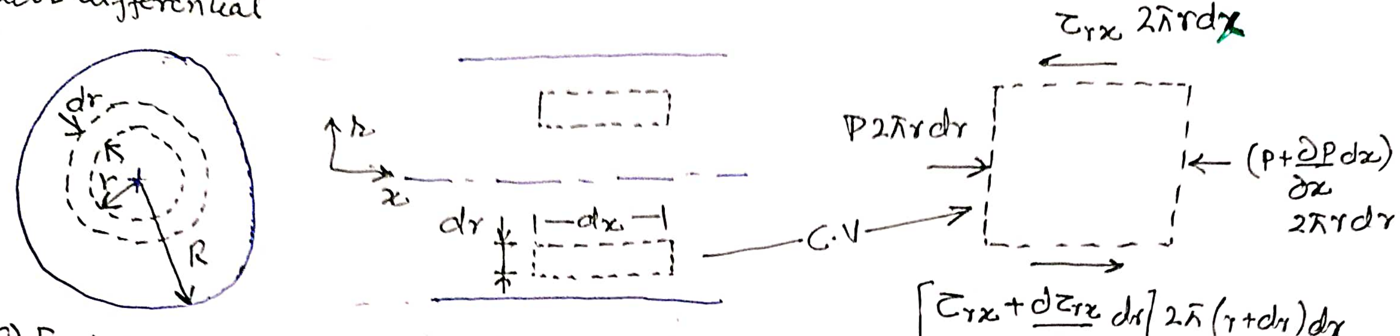
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \text{constt}$$





Fully Developed Laminar Flow

Annular differential C.V



a) End view of C.V. b) Side view of C.V

For a fully developed steady flow, the x-comp. of the momentum eq. is given by,

$$F_{s_x} = 0$$

or
$$-\frac{\partial p}{\partial x} 2\pi r dr dx + \tau_{rx} 2\pi dr dx + \frac{d\tau_{rx}}{dr} 2\pi r dr dx = 0$$

Dividing above eq. by $2\pi r dr dx$, we get.

$$\frac{\partial p}{\partial x} = \frac{\tau_{rx}}{r} + \frac{d\tau_{rx}}{dr} = \frac{1}{r}\frac{d(r\tau_{rx})}{dr}$$

The L.H.S of above eq. is at most a fn. of x. due to uniform pr. at each section. The R.H.S is at most a fn. of r only due to fully developed flow. Hence, the only way above eq can be valid for all x & r is for both sides to be equal to a constant.

$$\frac{1}{r}\frac{d(r\tau_{rx})}{dr} = \frac{\partial p}{\partial x} = \text{constt.} \quad \text{or} \quad \frac{d(r\tau_{rx})}{dr} = r\frac{\partial p}{\partial x}$$

Integrating above eq. we get.

$$r\tau_{rx} = \frac{r^2}{2}\left(\frac{\partial p}{\partial x}\right) + c_1$$

or
$$\tau_{rx} = \frac{r}{2}\left(\frac{\partial p}{\partial x}\right) + \frac{c_1}{r}$$

or
$$\mu\frac{du}{dr} = \frac{r}{2}\left(\frac{\partial p}{\partial x}\right) + \frac{c_1}{r} \qquad \tau_{rx} = \mu\frac{du}{dr}$$

and
$$u = \frac{r^2}{4\mu}\left(\frac{\partial p}{\partial x}\right) + \frac{c_1}{\mu}\ln r + c_2$$

For evaluatig c_1, & c_2, we know that $u=0$ at $r=R$ and at $r=0$ u is finite. Thus $c_1=0$. and previous eq. reduces to.

$$u = \frac{r^2}{4\mu}\left(\frac{\partial P}{\partial x}\right) + c_2$$

@ $r=R$ or $0 = \frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right) + c_2$ or $c_2 = -\frac{R^2}{4\mu}\frac{\partial P}{\partial x}$

$$\therefore u = \frac{r^2}{4\mu}\left(\frac{\partial P}{\partial x}\right) - \frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right) = -\frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$$

The shear stress:

$$\tau_{rx} = \mu\frac{du}{dr} = \frac{r}{2}\left(\frac{\partial P}{\partial x}\right)$$

The volume flow rate:

$$Q = \int_A \vec{V}\cdot d\vec{A} = \int_0^R u\,2\pi r\,dr = \int_0^R \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(r^2-R^2)\,2\pi r\,dr$$

or $Q = -\frac{\pi R^4}{8\mu}\left(\frac{\partial P}{\partial x}\right)$

In a fully developed flow, the pressure gradient $\frac{\partial P}{\partial x}$ is constant.

$$\therefore \frac{\partial P}{\partial x} = (P_2-P_1)/L = -\Delta P/L$$

Now, $Q = -\frac{\pi R^4}{8\mu}\left[\frac{-\Delta P}{L}\right] = \frac{\pi \Delta P R^4}{8\mu L} = \frac{\pi \Delta P D^4}{128\mu L}$

The avg. velo. magnitude:

$$\bar{V} = \frac{Q}{A} = \frac{Q}{\pi R^2} = -\frac{R^2}{8\mu}\left(\frac{\partial P}{\partial x}\right)$$

Point of maximum velocity:

To find that, we set $du/dr = 0$, and solve for r.

$$\frac{du}{dr} = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)r$$, Thus, $\frac{du}{dr}=0$ at $r=0$

At $r=0$, $u = u_{max} = U = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right) = 2\bar{V}$

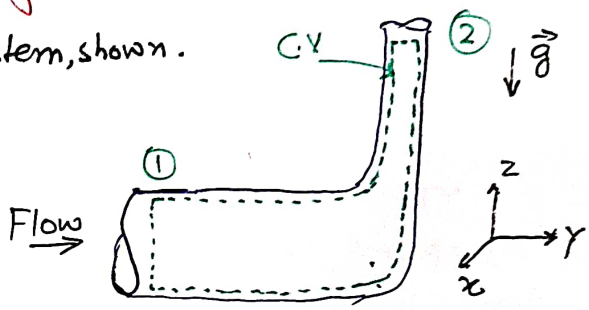
or $\boxed{\frac{u}{U} = 1-\left(\frac{r}{R}\right)^2}$

For a fully developed turbulent flow, $\frac{\bar{u}}{U} = \left(1-\frac{r}{R}\right)^{1/7}$

Energy Considerations in Pipe flow

Consider, steady flow through the piping system, shown.

The energy equation can be written as



$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall + \int_{CS} (e + pv)\rho \vec{V}\cdot d\vec{A}$$

where, $e = u + \frac{V^2}{2} + gz$.

Assumptions: 1) $\dot{W}_s$ = Shaft work = 0, $\dot{W}_{other} = 0$
2) $\dot{W}_{shear} = 0$ 3) Steady, 4) Incompressible flow. 5). Internal energy and pressure are uniform at ① & ②.

Under these assumptions, the energy equation reduces to.

$$\dot{Q} = \dot{m}(u_2 - u_1) + \dot{m}\left(\frac{p_2}{\rho} - \frac{p_1}{\rho}\right) + \dot{m}g(z_2 - z_1) + \int_{A_2} \frac{V_2^2}{2}\rho V_2 dA_2 - \int_{A_1} \frac{V_1^2}{2}\rho V_1 dA_1$$

In above eq. we have not assumed uniform velocity at ① & ②. In order to eliminate the integrals, we will introduce avg. velo. $(\bar{V})$. For this, we need to define kinetic energy coeff. (α), such that

$$\int_A \frac{V^2}{2}\rho V dA = \alpha \int_A \frac{\bar{V}^2}{2}\rho V dA = \alpha \dot{m}\frac{\bar{V}^2}{2}$$

or $\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2}$

α is the correction factor that allows us to use the average Velocity $\bar{V}$ in the energy eq. to compute the kinetic energy at a cross section.

For Laminar flow $\alpha = 2.0$.

For turbulent flow $\alpha = \left(\frac{U}{\bar{V}}\right)^3 \frac{2n^2}{(3+n)(3+2n)}$: for $n=7$, $\alpha \simeq 1$

Using α, the energy eq. can be written as.

$$\dot{Q} = \dot{m}(\overset{I.E}{u_2 - u_1}) + \dot{m}\left(\frac{P_2}{\rho} - \frac{P_1}{\rho}\right) + \dot{m}g(z_2 - z_1) + \dot{m}\left(\frac{\alpha_2 \bar{V}_2^2}{2} - \frac{\alpha_1 \bar{V}_1^2}{2}\right)$$

$$\text{or } \frac{\delta Q}{dm} = (u_2 - u_1) + \frac{P_2}{\rho} - \frac{P_1}{\rho} + gz_2 - gz_1 + \frac{\alpha_2 \bar{V}_2^2}{2} - \frac{\alpha_1 \bar{V}_1^2}{2}$$

$$\text{or } \underbrace{\left(\frac{P_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right)}_{\text{Mechanical energy/mass at a cross-section}} - \left(\frac{P_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = \underbrace{\overbrace{(u_2 - u_1)}^{\text{unwanted thermal energy}} - \frac{\delta Q}{dm}}_{\text{}}$$

$\frac{\delta Q}{dm}$ — loss of energy via H.T

H.T: Heat Transfer

Diff. in Mechanical energy per unit mass between sections ① & ② = Total energy loss/mass.

$$\text{or } \left(\frac{P_1}{\rho} + \frac{\alpha_1 \bar{V}_1^2}{2} + gz_1\right) - \left(\frac{P_2}{\rho} + \frac{\alpha_2 \bar{V}_2^2}{2} + gz_2\right) = h_{lT} \quad \text{Units: } L^2/t^2 \quad \text{—(A)}$$

$$\text{or } \left(\frac{P_1}{\rho g} + \frac{\alpha_1 \bar{V}_1^2}{2g} + z_1\right) - \left(\frac{P_2}{\rho g} + \frac{\alpha_2 \bar{V}_2^2}{2g} + z_2\right) = \frac{h_{lT}}{g} \quad \text{Units} = L \quad \text{—(B)}$$

Losses in Pipe flow:

Total head Loss, h_{lT}, is regarded as the sum of major losses, h_{l_m} due to frictional effects in fully developed flow in constant area tubes, and minor losses h_{lm}, resulting from entrances, fittings, area changes, etc.

Major Losses.

Major head loss, h_l can be evaluated from the energy balance eq. (A)

For a fully developed flow through a horizontal, constant area pipe, $h_{lm} = 0$ and $\alpha_1 \bar{V}_1^2/2 = \alpha_2 \bar{V}_2^2/2$

$$\therefore \frac{P_1 - P_2}{\rho} = \frac{\Delta P}{\rho} = h_l$$

Laminar flow:

For Laminar flow $\Delta p = \frac{128 \mu L Q}{\pi D^4} = \frac{128 \mu L \bar{V}(\pi D^2/4)}{\pi D^4} = 32 \frac{L}{D}\frac{\mu \bar{V}}{D}$

$$\therefore \quad h_l = 32 \frac{L}{D}\frac{\mu \bar{V}}{\rho D} = \frac{L}{D}\frac{\bar{V}^2}{2}\left(64 \frac{\mu}{\rho \bar{V} D}\right) = \left(\frac{64}{Re}\right)\frac{L}{D}\frac{\bar{V}^2}{2}$$

Turbulent flow:

For turbulent flow, we donot have analytical expression for h_l.

Experimental results along with Dimensional analysis is used.

In a fully developed turbulent flow,

$$\Delta p = \Delta p(D, L, e, \bar{V}, \rho, \mu)$$ where e: pipe roughness.

Applying dimensional analysis, we obtain.

$$\frac{\Delta p}{\rho \bar{V}^2} = f_1\left(\frac{\mu}{\rho \bar{V} D}, \frac{L}{D}, \frac{e}{D}\right)$$

or $$\frac{h_l}{\bar{V}^2} = \phi\left(Re, \frac{L}{D}, \frac{e}{D}\right)$$

Experiments show that $h_l \propto L/D$

$$\therefore \quad \frac{h_l}{\bar{V}^2} = \frac{L}{D}\phi_1\left(Re, \frac{e}{D}\right)$$

or $$\frac{h_l}{\frac{1}{2}\bar{V}^2} = \frac{L}{D}\phi_2\left(Re, \frac{e}{D}\right)$$ In this form L.H.S represents the ratio of the head loss to the kinetic energy per unit mass of flow.

The function $\phi_2(Re, e/D)$, is known as the friction factor, f

$$f \equiv \phi_2(Re, e/D).$$

and, $$h_l = f \frac{L}{D}\frac{\bar{V}^2}{2} \quad \text{or} \quad H_l = f \frac{L}{D}\frac{\bar{V}^2}{2g}$$

The friction factor f, is determined experimentally. One such study performed by L.F. Moody. is shown here.

P.S: Show fig: 6·13 (F.M. White 4e)

From the curve, f, can be found at known Re and e/D and then head loss can be found.

Friction factor for laminar flow, can be found by comparing.

$$h_l = \left(\frac{64}{Re}\right)\frac{L}{D}\frac{\bar{V}^2}{2} = f\,\frac{L}{D}\frac{\bar{V}^2}{2}$$

Thus $f_{Lam} = \dfrac{64}{Re}$: For laminar flow f is a fn of Re only.

Various mathematical expressions have been fitted to the data to avoid using graphical method for obtaining f for turbulent flows.

Colebrook: $$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Haaland: $$\frac{1}{\sqrt{f}} = -1.8\log\left(\left(\frac{e/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)$$

Blasius correlation: $f = \dfrac{0.316}{Re^{0.25}}$: Valid for $Re \leq 10^5$

Minor Losses.

Flow in pipes may go through various fittings, bends, or abrupt changes in area. Additional head losses occur, mainly due to flow separation. In separated zones, energy is dissipated due to violent mixing. These losses are minor if the piping system includes long lengths of constant area-pipes. The minor loss is given by,

$$h_{lm} = K\frac{\bar{V}^2}{2}.$$ where, K = loss coefficient.

K, is determined experimentally for each situation.

OR, $h_{lm} = f\dfrac{L_e}{D}\dfrac{\bar{V}^2}{2}$; L_e is an equivalent length of straight pipe.

a) Inlets & Exits.

A poorly designed inlet to a pipe can cause appreciable head loss. Flow separation occurs if inlet has sharp corners. A vena contracta is formed, wherein the flow must accelerate locally to pass through the reduced flow area at vena contracta. Losses in mechanical energy result from the unconfined mixing as the flow stream decelerates again to fill the pipe. No improvement in minor loss coeff for an exit is possible; however, addition of a diffuser can reduce $\bar{V}^2/2$ and therefore h_{lm} considerably.

P.S: Show fig: 6.21 (F.M. White) 4e.

b) Sudden expansion & sudden contraction.

P.S: Show fig 6.22 (F.M. White 4e)

Both loss coeff. are based on the larger $\bar{V}^2/2$. (Velo. in smaller area).
Losses caused by area change can be reduced by using a nozzle or a diffuser b/w two sections of straight pipe. The loss coeff (K) data for nozzles for different area ratio & conical angle can be found in Literature (ASHRAE Handbook – Fundamentals.)
Losses in diffuser depend on a number of geometric and flow variables. Diffuser data is presented in terms of a pressure recovery coeff, C_P, defined as the ratio of static pressure rise to inlet dynamic pr.

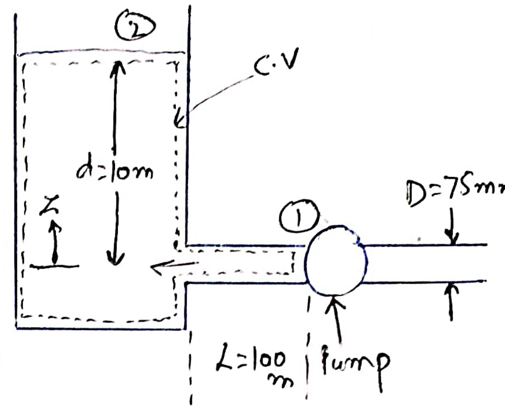
$$C_P \equiv \frac{P_2 - P_1}{\frac{1}{2}\rho \bar{V}_1^2}$$

P.S: Show fig 6.23 (F.M. White 4e)

& $$K = \frac{h_{lm}}{\bar{V}_1^2/(2g)} = \boxed{1 - \frac{d_1^2}{d_2^4} - C_P}$$

P.S: Read yourself: c) Losses due to bends. & d) Losses due to pipe fittings.

Prob: Water is pumped at $0.01\, m^3/s$ through a 75 mm dia. smooth pipe of length $L = 100$ m, into a const-level reservoir of depth $d = 10$ m. Find the pump pr, P_1, required to maintain the flow.



Soln:

$$\left(\frac{P_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{P_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{L_T} = h_L + h_{lm} \quad \text{—— (1)}$$

where $h_L = f\frac{L}{D}\frac{\bar{V}^2}{2}$ and $h_{lm} = K\frac{\bar{V}^2}{2}$

Here, $P_1 = P_{pump}$, $P_2 = 0$ (guage), $\Delta p = p_1 - p_2 = P_{pump}$; $\bar{V}_1 = \bar{V}$, $\bar{V}_2 \simeq 0$, K(exit loss) $= 1.0$, and $\alpha_1 \simeq 1.0$, if $z_1 = 0$, $z_2 = d$.

Now eq (1) becomes.

$$\frac{\Delta P}{\rho} + \frac{\bar{V}^2}{2} - gd = f\frac{L}{D}\frac{\bar{V}^2}{2} + \frac{\bar{V}^2}{2}$$

or $P_{pump} = \Delta p = \rho\left(gd + f\frac{L}{D}\frac{\bar{V}^2}{2}\right)$; $\bar{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = 2.26\, m/s$

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{999 \times 2.26 \times 0.075}{1 \times 10^{-3}} = 1.7 \times 10^5$$

For turbulent flow in a smooth pipe $(e = 0)$. using Colebrook relation.

$$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \; ; \quad f = 0.0162.$$

$\therefore$ $P_{pump} = 1.53 \times 10^5\ N/m^2$ (guage)

Pumps, Fans and Blowers in Fluid Systems.

Neglecting H.T and internal energy changes of the fluid, the Ist Law of thermodynamics applied ~~to~~ across the pump is.

$$\dot{W}_{pump} = \dot{m}\left[\left(\frac{P}{\rho}+\frac{\bar{V}^2}{2}+gz\right)_{discharge} - \left(\frac{P}{\rho}+\frac{\bar{V}^2}{2}+gz\right)_{suction}\right]$$

The head Δh_{pump} produced by the pump in (energy/mass) units, is.

$$\Delta h_{pump} = \frac{\dot{W}_{pump}}{\dot{m}} \quad — (1)$$

In many cases, inlet and outlet pipe diameters and ~~same~~ elevations are same or negligibly different.

$$\therefore \Delta h_{pump} = \frac{\Delta P_{pump}}{\rho} \quad — (2)$$

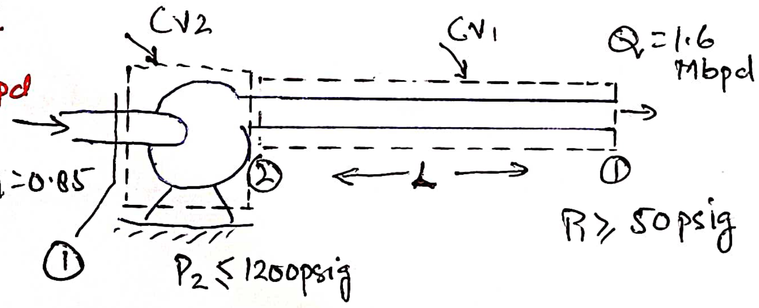
\# A pump adds energy to the fluid in the form of a gain in pressure.

Using eq (1), (2) along with $\dot{m} = \rho Q$ we can say.

$$\dot{W}_{pump} = Q\,\Delta P_{pump}$$

Pump efficiency $\eta = \dfrac{\dot{W}_{pump}}{\dot{W}_{in}}$ → power reaching the fluid; → power input to the pump.

Prob. Crude oil flows through a level section of a pipeline at $Q = 1.6$ Mbpd Million barrels per day (1 barrel = 42 gal). Inside pipe dia = 4 in; its roughness is equivalent to galvanised iron.



Maximum allowable pr is 1200 psig. The minimum pr. required to keep dissolved gases in solution in oil is 50 psig. SG = of crude oil = 0.93. μ at 140°F = 3.5×10^{-4} lbf·s/ft². Find maximum possible spacing b/w pumping stations. If η_{pump} is 85%, determine the power that must be supplied at each station.

Soln: CV1 is for pipe flow from ② to ①. CV2 is for pump (① to ②).

Applying the energy eq. for steady, incompressible pipe flow to CV1.

$$\left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) - \left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) = h_{lT} = h_l + h_{lm}.$$

Assumptions: 1) $\alpha_1\bar{V}_1^2 = \alpha_2\bar{V}_2^2$, 2) $z_1 = z_2$, 3) $h_{lm} = 0$ 4, Const viscosity.

Now,
$$\Delta p = p_2 - p_1 = f\frac{L}{D}\,\rho\frac{\bar{V}^2}{2}$$

or
$$L = \frac{2D}{f}\,\frac{\Delta p}{\rho\bar{V}^2}$$

Now. $Q = 1.6\times10^6 \frac{bbl}{day} \times 42\frac{gal}{bbl} \times \frac{ft^3}{7.48\,gal} \times \frac{day}{24\,hr} \times \frac{hr}{3600\,s} = 104\,ft^3/s$

So $\bar{V} = \frac{Q}{A} = \frac{104}{\pi(4)^2/4} = 8.27\,ft/s$; $Re = \frac{\rho\bar{V}D}{\mu} = (0.93)1.94\frac{slug}{ft^3} \times 8.27\frac{ft}{s} \times 4\,ft \times \frac{ft^2}{3.5\times10^{-4}}$

$= 1.71\times10^5$

From literature $e = 0.0005\,ft$; $e/D = 0.00012$. Using Colebrook relation we get $f = 0.017$.

$\therefore L = \frac{2}{0.017} \times 4\,ft \times (1200-50)\frac{lbf}{in^2} \times \frac{ft^3}{(0.93)(1.94)\,slug} \times \frac{s^2}{(8.27)^2 ft^2} \times 144\frac{in^2}{ft^2} \times \frac{slug\cdot ft}{lbf\cdot s^2}$

$= 6.32\times10^5\,ft$

Applying I[st] Law of thermodynamics to CV_2.

$$\dot{W}_{pump} = Q\Delta p_{pump} = 104\frac{ft^3}{s} \times \frac{(1200-50)\,lbf}{in^2} \times \frac{144\,in^2}{ft^2} \times \frac{hp\cdot s}{550\,ft\cdot lbf}$$

$\simeq 31{,}300\,hp$

also
$$\eta = \frac{\dot{W}_{pump}}{\dot{W}_{in}}$$

$$\therefore \dot{W}_{in} = \frac{31300}{0.85} = 36800\,hp.$$

$$\left(\frac{P_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)-\left(\frac{P_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)=h_{lT}=h_l+h_{lm}.$$

Assumptions: 1) $\alpha_1\bar{V}_1^2=\alpha_2\bar{V}_2^2$, 2) $z_1=z_2$, 3) $h_{lm}=0$ 4, Const viscosity.

Now, $\Delta P=P_2-P_1=f\frac{L}{D}\,\rho\frac{\bar{V}^2}{2}$

or $L=\frac{2D}{f}\frac{\Delta P}{\rho\bar{V}^2}$.

Now, $Q=1.6\times10^6\,\frac{bbl}{day}\times 42\,\frac{gal}{bbl}\times\frac{ft^3}{7.48\,gal}\times\frac{day}{24\,hr}\times\frac{hr}{3600\,s}=104\,ft^3/s$

So $\bar{V}=\frac{Q}{A}=\frac{104}{\frac{\pi(4)^2}{4}}=8.27\,ft/s$; $Re=\frac{\rho\bar{V}D}{\mu}=(0.93)1.94\,\frac{slug}{ft^3}\times8.27\,\frac{ft}{s}\times4\,ft\times\frac{ft^2}{3.5\times10^{-4}}$

$=1.71\times10^5$

From literature $e=0.0005\,ft$; $e/D=0.00012$. Using Colebrook relation we get $f=0.017$.

$\therefore L=\frac{2}{0.017}\times4\,ft\times(1200-50)\frac{Lbf}{in^2}\times\frac{ft^3}{(0.93)(1.94)\,slug}\times\frac{s^2}{(8.27)^2\,ft^2}\times144\,\frac{in^2}{ft^2}\times\frac{slug\,ft}{lbf\cdot s^2}$

$=6.32\times10^5\,ft$

Applying Ist Law of thermodynamics to CV_2.

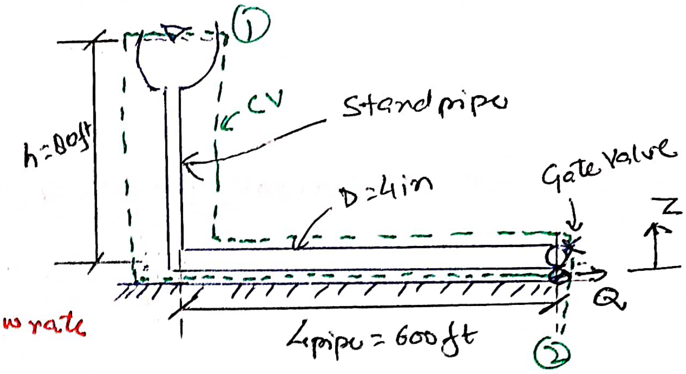
$\dot{W}_{pump}=Q\Delta P_{pump}=104\,\frac{ft^3}{s}\times\frac{(1200-50)\,lbf}{in^2}\times144\,\frac{in^2}{ft^2}\times\frac{hp\cdot s}{550\,ft\cdot lbf}$

$\approx 31{,}300\,hp.$

also $\eta=\frac{\dot{W}_{pump}}{\dot{W}_{in}}$

$\therefore\ \dot{W}_{in}=\frac{31300}{0.85}\approx 36800\,hp.$

Prob. A fire protection system is shown. The longest pipe in the system is 600 ft. and is made of cast iron 20 yr old. Pipe contains one gate valve; neglect other minor losses. Determine the max. flow rate through this pipe.



Soln: Assumptions: $p_1 = p_2 = p_{atm}$, 2) $\bar{V}_1 = 0$, $\alpha_2 \simeq 1.0$

Gov. eq. $$\left(\cancel{\frac{p_1}{\rho}} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\cancel{\frac{p_2}{\rho}} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{lT} = h_l + h_{lm}$$

$$\therefore \quad g(z_1 - z_2) - \frac{\bar{V}_2^2}{2} = h_{lT} = f\left(\frac{L}{D} + \frac{L_e}{D}\right)\frac{\bar{V}_2^2}{2} \qquad \left[h_{lm} = f\frac{L_e}{D}\frac{\bar{V}^2}{2}\right]$$

For a fully open gate valve. $L_e/D = 8$.

$$g(z_1 - z_2) = \frac{\bar{V}_2^2}{2}\left[f\left(\frac{L}{D} + 8\right) + 1\right]$$

or $$\bar{V}_2 = \left[\frac{2g(z_1 - z_2)}{f(L/D + 8) + 1}\right]^{1/2} \qquad \text{—— (A)}$$

Assume the stand pipe is of same dia. as the horizontal pipe.

$$\therefore \quad \frac{L}{D} = \frac{(600 + 80)\,ft}{4\ in} \times \frac{12\ in}{ft} = 2040.$$

$$z_1 - z_2 = h = 80\,ft.$$

To solve eq (A) we need to iterate. For that we need to make an estimate for f by assuming fully turbulent flow (where f is constt). This value can by obtained by solving Colebrook correlation. For large value of Re ($\sim 10^8$) and $e/D \simeq 0.005$ ($e = 0.00085$ for cast iron and doubled to allow for old pipe). we find $f \simeq 0.03$. Thus first value of $\bar{V}_2$ is

$$\bar{V}_2 = \left[2 \times 32.2\,\frac{ft}{s^2} \times 80\,ft \times \frac{1}{0.03(2040+8)+1}\right]^{1/2} = 9.08\ ft/s$$

4.13

Now, new value of f:

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = 9.08 \frac{ft}{s} \times \frac{ft}{3} \times \frac{s}{1.21 \times 10^{-5} ft^2} = 2.5 \times 10^5$$

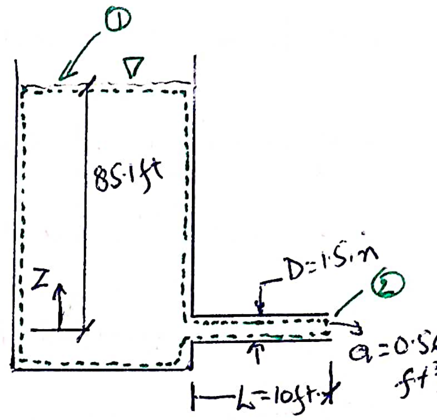
For $e/D = 0.005$, $f = 0.0308$, using Colebrook relation. Now

$$\bar{V}_2 = \left[2 \times 32.2 \frac{ft}{s^2} \times 80\, ft \times \frac{1}{0.0308\,(2040 + 8) + 1} \right]^{1/2} = 8.97\, ft/s$$

8.97 ~~ ~~ differs from 9.08 by less than 2%. therefore acceptable.

$$Q = \bar{V}_2 A = 8.97 \frac{ft}{s} \times \frac{\pi}{4} \left(\frac{1}{3}\right)^2 ft^2 \times 7.48 \frac{gal}{ft^3} \times 60 \frac{s}{min} = 351\, gpm.$$

Prob. A copper pipe 10 ft long, with 1.5 in i.d was used to determine entrance losses. Pipe discharges to atmosphere. For a square-edged entrance, a discharge of 0.566 ft³/s was measured when the reservoir level was 85.1 ft. Evaluate the loss coefficient.



Solu: Assumptions: 1) $P_1 = P_2 = P_{atm}$; 2) $\bar{V}_1 \approx 0$

Gov. eq. $$\cancel{\frac{P_1}{\rho}} + \cancel{\alpha_1 \frac{\bar{V}_1^2}{2}} + gz_1 = \cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{\bar{V}_2^2}{2} + \cancel{gz_2} + h_{LT}$$

$$h_{LT} = f \frac{L}{D} \frac{\bar{V}_2^2}{2} + K_{en} \frac{\bar{V}_2^2}{2}$$

or $$Z_1 = h = \alpha_2 \frac{\bar{V}_2^2}{2g} + f \frac{L}{D} \frac{\bar{V}_2^2}{2g} + K_{en} \frac{\bar{V}_2^2}{2g}$$

or $$K_{en} = \frac{2gh}{\bar{V}_2^2} - f\frac{L}{D} - \alpha_2$$

Now, $$\bar{V}_2 = \frac{Q}{A} = \frac{4}{\pi} \times 0.566 \frac{ft^3}{s} \times \frac{1}{(1.5)^2 in^2} \times 144 \frac{in^2}{ft^2} = 46.1\, ft/s$$

Assuming $T = 70°F$, $\nu = 1.05 \times 10^{-5}\, ft^2/s$.

$$Re = \frac{\bar{V}D}{\nu} = 46.1 \times \frac{ft}{s} \times 1.5\, in \times \frac{s}{1.05 \times 10^{-5} ft^2} \times \frac{ft}{12\, in} = 5.49 \times 10^5$$

For drawn tubing, $e = 5 \times 10^{-6}\, ft$, so $e/D = 0.00004$ and $f = 0.0135$ [Colebrook]

Instead of assuming $\alpha \approx 1$ like before, we will compute it as K_{en} depends significantly on it. Using the relation for α.

$$\alpha = \left(\frac{U}{\bar{V}}\right)^3 \frac{2n^2}{(3+n)(3+2n)}$$

Also, $n = -1.7 + 1.8 \log(Re_U) \approx 8.63$ — we assumed $Re_U \approx Re_{\bar{V}}$

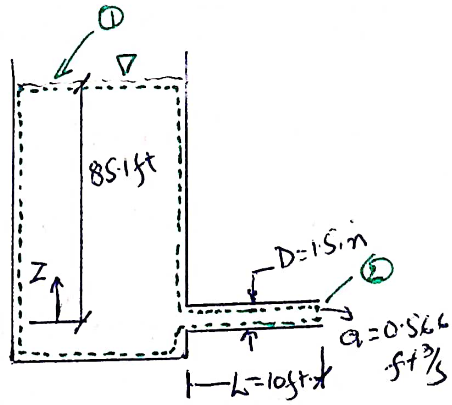
$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} = 0.847$$

$\alpha = 1.04$

$$K_{en} = 2 \times 32.2 \frac{ft}{s^2} \times 85.1 ft \times \frac{s^2}{(46.1)^2 ft^2} - 0.0135 \frac{10 ft}{1.5 in} \times \frac{12 in}{ft} - 1.04$$

$= 0.459.$

Prob. A copper pipe 10 ft long, with 1.5 in i.d was used to determine entrance losses. Pipe discharges to atmosphere. For a square-edged entrance, a discharge of 0.566 ft³/s was measured when the reservoir level was 85.1 ft. Evaluate the loss coefficient.



Solu: Assumptions: 1) $p_1 = p_2 = p_{atm}$; 2) $\bar{V}_1 \approx 0$

Gov. eq. $\cancel{\frac{p_1}{\rho}} + \cancel{\alpha_1 \frac{\bar{V}_1^2}{2}} + g z_1 = \cancel{\frac{p_2}{\rho}} + \alpha_2 \frac{\bar{V}_2^2}{2} + \cancel{g z_2} + h_{LT}$

$$h_{LT} = f \frac{L}{D} \frac{\bar{V}_2^2}{2} + K_{en} \frac{\bar{V}_2^2}{2}$$

or $$z_1 = h = \alpha_2 \frac{\bar{V}_2^2}{2g} + f \frac{L}{D} \frac{\bar{V}_2^2}{2g} + K_{en} \frac{\bar{V}_2^2}{2g}$$

or $$K_{en} = \frac{2gh}{\bar{V}_2^2} - f\frac{L}{D} - \alpha_2$$

Now, $\bar{V}_2 = \frac{Q}{A} = \frac{4}{\pi} \times 0.566 \frac{ft^3}{s} \times \frac{1}{(1.5)^2 in^2} \times 144 \frac{in^2}{ft^2} = 46.1 ft/s$

Assuming $T = 70°F$, $\nu = 1.05 \times 10^{-5} ft^2/s$.

$$Re = \frac{\bar{V}D}{\nu} = 46.1 \times \frac{ft}{s} \times 1.5 in \times \frac{s}{1.05 \times 10^{-5} ft^2} \times \frac{ft}{12 in} = 5.49 \times 10^5$$

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$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} = 0.847$$

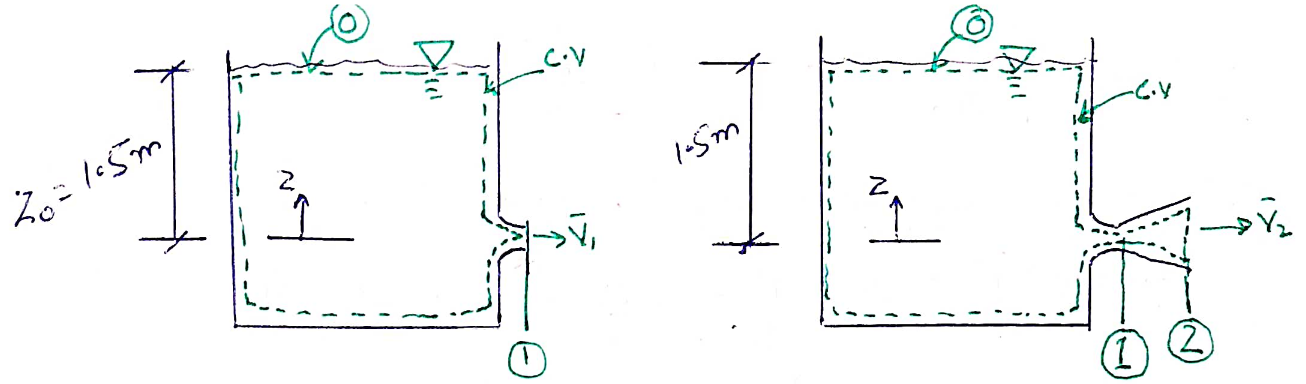
$\alpha = 1.04$

$$K_{en} = 2 \times 32.2 \frac{ft}{s^2} \times 85.1 ft \times \frac{s^2}{(46.1)^2 ft^2} - 0.0135 \frac{10 ft}{1.5 in} \times \frac{12 in}{ft} - 1.04$$

$= 0.459.$

The large head loss in a square-edged entrance is primarily due to separation at the sharp inlet corner and formation of a vena contracta immedeatly downstream from the corner. The effective flow area reaches a minimum at the vena contracta, so the flow velocity is maximum there. The flow expands again following the vena contracta to fill the pipe. The uncontrolled expansion following the vena contracta is responsible for most of the head loss. Rounding the inlet corner reduces the extent of separation significantly. This reduces the velo. increase through the vena contracta and consequently reduces the head loss. A well rounded inlet almost eliminates flow separation.

P. S: Flow through multiple path system.



Prob. A circular tubular bronze nozzle is attached to a tank. Due to this nozzle the flow rate is fixed. To increase the discharge a diffuser is attached on the outlet of the nozzle. Nozzle exit dia = 25mm. Determine increase in flow rate when a diffuser with N/R1 = 3.0 and AR = 2.0 is attached.

Solu: Assumptions: (1) $\bar{V}_0 \simeq 0$ (2) $\alpha_1 \simeq 1$

For nozzle alone,

$$\cancel{\frac{p_0}{\rho}} + \cancel{\alpha_0 \frac{\bar{V}_0^2}{2}} + gz_0 = \cancel{\frac{p_1}{\rho}} + \alpha_1 \frac{\bar{V}_1^2}{2} + \cancel{gz_1} + h_{LT}$$

$$h_{LT} = K_{en} \frac{\bar{V}_1^2}{2}$$

Thus $gz_0 = \frac{\bar{V}_1^2}{2} + K_{en}\frac{\bar{V}_1^2}{2} = (1+K_{en})\frac{\bar{V}_1^2}{2}$

Solving for $\bar{V}_1$ and using $K_{en} \simeq 0.04$.

$$\bar{V}_1 = \sqrt{\frac{2gz_0}{1.04}} = \sqrt{\frac{2}{1.04} \times 9.81 \times 1.5} = 5.32 \text{ m/s}$$

$$Q = \bar{V}_1 A_1 = \bar{V}_1 \frac{\pi D_1^2}{4} = 5.32 \times \frac{\pi}{4}(0.025)^2 = 0.00261 \text{ m}^3/\text{s}$$

For nozzle with diffuser attached.

$$\cancel{\frac{p_0}{\rho}} + \cancel{\alpha_0 \frac{\bar{V}_0^2}{2}} + gz_0 - \left[\cancel{\frac{p_2}{\rho}} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right] = h_{LT}$$

$$\text{or } h_{LT} = K_{en}\frac{\bar{V}_1^2}{2} + K_{diff}\frac{\bar{V}_1^2}{2}$$

$$\text{or } gz_0 = \frac{\bar{V}_2^2}{2} + (K_{en} + K_{diff})\frac{\bar{V}_1^2}{2}$$

From continuity $\bar{V}_1 A_1 = \bar{V}_2 A_2$, so, $\bar{V}_2 = \bar{V}_1 \frac{A_1}{A_2} = \bar{V}_1 \frac{1}{AR}$

Elev. = 60m

Elevation = 20m

4.17

or $gz_0 = \left[\frac{1}{(AR)^2} + K_{en} + K_{diff}\right]\frac{\bar{V}_1^2}{2}$

To obtain K_{diff} apply the energy eq from ① to ②

$$\frac{P_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2 + K_{diff}\frac{\bar{V}_1^2}{2}$$

Solving with $\alpha_2 \simeq 1$, we get

$$K_{diff} = 1 - \frac{\bar{V}_2^2}{\bar{V}_1^2} - \boxed{\frac{P_2 - P_1}{\frac{1}{2}\rho\bar{V}_1^2}} = 1 - \left(\frac{A_1}{A_2}\right)^2 - C_p = 1 - \frac{1}{(AR)^2} - C_p$$

where C_p = Pressure recovery coefficient

For $C_p = 0.45$ (based on $N/R_1 = 3.0$ & $AR = 2$)

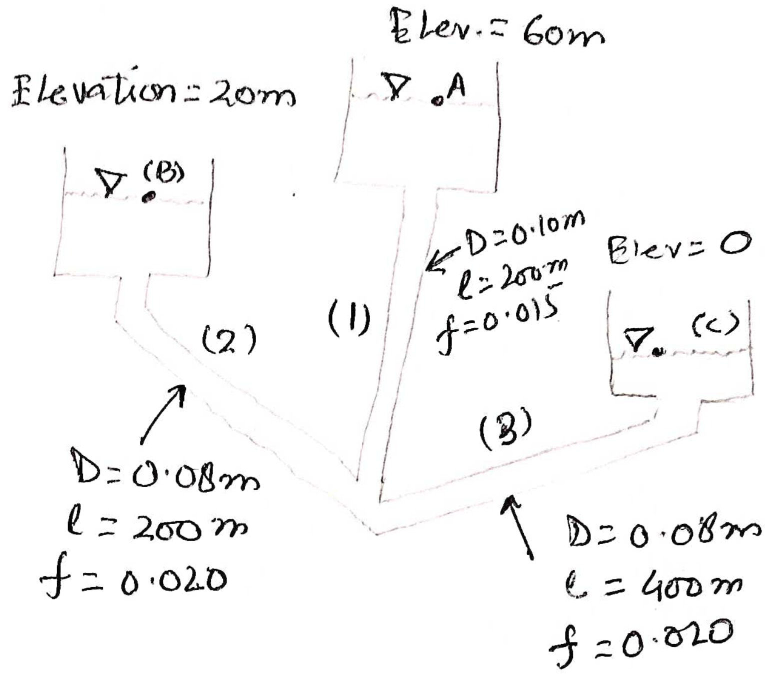
$$K_{diff} = 1 - \frac{1}{(2)^2} - 0.45 = 0.3$$

$$\bar{V}_1 = \sqrt{\frac{2gz_0}{0.25 + 0.04 + 0.3}} = \sqrt{\frac{2}{0.59} \times 9.81 \times 1.5} = 7.06 \text{ m/s}$$

& $Q_d = \bar{V}_1 A_1 = 7.06 \times \frac{\pi}{4}(0.025)^2 = 0.00347 \text{ m}^3/\text{s}$

Flow rate increase from adding diffuser is

$$\frac{\Delta Q}{Q} = \frac{Q_d - Q}{Q} = \frac{Q_d}{Q} - 1 = \frac{0.00347}{0.00261} - 1 = 0.33 \text{ or } 33\%$$


Elev. = 60m
A
Elevation = 20m
(B)
D = 0.10m
l = 200m
f = 0.015
Elev = 0
(C)
(1)
(2)
(3)
D = 0.08m
l = 200m
f = 0.020
D = 0.08m
l = 400m
f = 0.020